A beginning is the time for taking the most delicate care that the balances are correct.

— Frank Herbert, Dune

1 Tools of the Trade

1.1 Keeping it simple

The structure of the universe is both subtle and complex. You could spend your entire life studying it[1] and never fully figure it out. But you need not despair, since there are a great many things which we do understand quite well, and some of these we will study here. One way to tackle difficult physics is to use simplified models of real-world objects.

This Course	The Real World
All objects fall at the same rate.	The rate of an object's fall depends on its shape and density.
Massless strings, springs and pulleys.	All of these things may be light, but they are never massless.
Frictionless surfaces.	Surfaces can have very low friction, but are never frictionless.
Friction is simple and depends only on the materials involved.	Friction is complicated and depends on materials involved, surface smoothness, oils, temperature, humidity...
A ball is a perfect sphere.	A ball is never perfectly spherical.[2]
Copper has a density of $8,960 \text{ kg/m}^3$.	The density of copper varies from $8,928 - 8,993 \text{ kg/m}^3$.

If physics is supposed to provide an accurate description of the universe, why make such simplifications? The main reason is that these simplifications make it much easier to do physics. Learning to think like a physicist takes time, and the best way to develop your physics skills is to start simple. Another reason is that these simplifications are not as terrible as you might think. We can use them and still yield answers which are very close to reality. Learning about simple physical models will give you a deeper understanding of how the universe works.

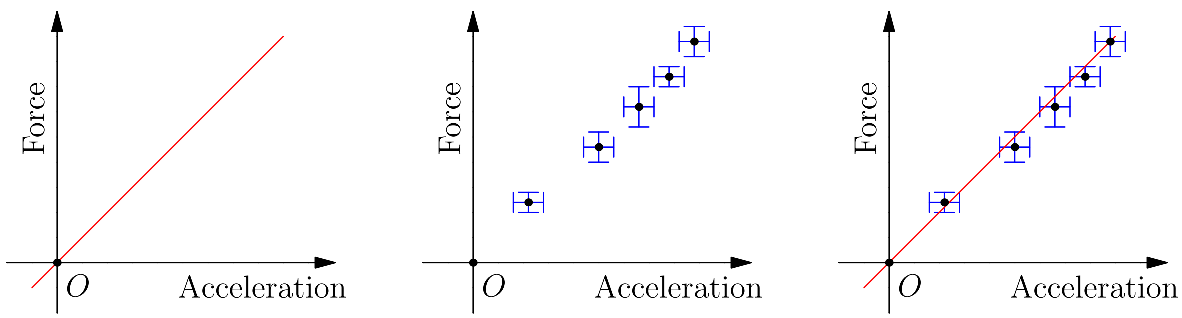
[1] Some of us have!

[2] A perfect sphere would be quite useful. Scientists in Australia have been working very hard to make a nearly perfect sphere in order have a more precise definition of the kilogram. You can read about their work at `http://www.theage.com.au/news/national/making-an-exact-difference/2007/06/14/1181414466901.html`

Still, we must be careful not to confuse our simple models with reality. One of the skills you will learn is when to simplify a problem, and when to keep complexities. As we progress through the course we will simplify less, but we will always be making simplifications. There will be times when I will gloss over details[3] in order to keep our models simple, but I will try to tell you when I do.

1.2 Keeping it real

Physics is more than just mathematics and philosophy. It is grounded in reality. The fundamental tenet of science is that it must be tested and verified. Any model which fails to conform to reality is rejected. Much of the physics we will study is hundreds of years old, and has been verified scientifically countless times. It really works.



(a) The Theory (b) The Experiment (c) Testing the theory

Scientific models make predictions which can be tested. The experiments we will do in workshop will not simply confirm what you already know, they will test our models from lecture. It is the development of useful models, confirmed by experiment, which makes physics useful and powerful. This course will develop your skills in both theoretical and experimental techniques.

1.3 Units

Measurements always require a standard of comparison. I have measured the length of my desk to be 12 span, but this measurement is meaningless without a formal definition for the length of a span.[4] So we need a set of standards for mass, length, and time.[5] Ideally these standards should be determined by measurements you can take anywhere, rather than based on a specific object. This is true for length and time, but not yet for mass.

The standards we will use are those of the International System, also known as the metric system. They are defined as follows:

[3] Also called lying by omission.

[4] A span is defined as the distance from the tip of the thumb to the tip of the pinky of a splayed hand. Formally, it is equal to 9 inches.

[5] For now we only need these three. In general there are seven base standards. The remaining four include electric current, temperature, quantity, and luminosity.

Second (time) The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

Meter (length) The meter is the length of the path travelled by light during a time interval of 1/299, 729, 458 of a second.

Kilogram (mass) The kilogram is equal to the mass of the international prototype of the kilogram.

While these definitions are very precise, they don't really give us a feel for them in practical terms. In very rough terms, a second is about the time between heartbeats. If you feel your pulse when you are rested, your heartbeats are about a second apart. A meter is roughly the distance between the tip of your nose and the tips of your fingers when your arm is outstretched. A kilogram has a weight on earth of a liter of water, so a two-liter bottle of soda has a mass of roughly two kilograms. When doing experiments we will use precise instruments, but it is useful to have a practical feel for these units as well.

1.4 Requisite Mathematics

Physics is a mathematical science. As such, we will be using mathematics extensively. You are expected to be comfortable with a range of skills. These include

Arithmetic You should be intimately familiar with basic operations $(+, -, \times, \div)$, as well as the order of operations. You should understand your calculator and know how to use it effectively. Given

$$x = \left[\frac{\sqrt[4]{85/(15-6.2)}}{8.8} \right]^{0.3}$$

you should have no difficulty determining x.[6]

Algebra You should know how to simplify algebraic equations. You should know how to use the quadratic equation. Given

$$y^2 - 6y + 8 = 0$$

You should be able to determine y. You should also know how to solve simultaneous equations. Given

$$\begin{aligned} 2x + 5y &= 2.75 \\ 6x + 2y &= 2.10 \end{aligned}$$

you should be able to solve for x and y. This will be very useful for projectile motion problems.

[6] $x = 0.6173$. Did you get it right?

Trigonometry You must know the meaning and forms of $\sin x$, $\cos y$, $\tan z$. You should know the largest and smallest values of these functions, and the angle(s) at which they occur. You should also understand trigonometric identities and know where to find them. You should be able to simplify expressions such as

$$\sin(\alpha + \beta)$$

You do not have to memorize trigonometric identities, but you should know where to find them.

Calculus You should be able to differentiate functions such as

$$y = x^2 + 3$$
$$y = \ln(x)$$
$$y = x^3 \sin(14x)$$

You should also be able to integrate various functions:

$$\int \left(5x^3 - 6x^2 + 12\right) \mathrm{d}x$$
$$\int \frac{1}{12} \sin(8x) \, \mathrm{d}x$$
$$\int e^{x/2} \, \mathrm{d}x$$

It will be assumed that you already possess these mathematical skills, therefore we will not go over them in detail. If you are weak in any of these areas, it is your responsibility to strengthen them.

1.5 Approximations

There are times when it will be useful to make mathematical approximations in order to simplify our calculations. A good example of this is gravitational acceleration. Near the Earth the gravitational acceleration of any object is a constant $g = 9.8$ m/s^2. In reality, gravity is not so simple.[7] A more accurate description of gravity is given by

$$g = \frac{GM_E}{r^2}$$

where

G is a constant
M_E is the mass of the Earth
r is the distance from the center of the Earth

[7] Not even remotely. Ask me about my doctoral thesis sometime.

At the surface of the Earth, this acceleration is[8]

$$g = \frac{GM_E}{{R_E}^2} = 9.8\ \text{m/s}^2$$

however at a distance h above the surface of the earth, the gravitational acceleration is

$$g' = \frac{GM_E}{(R_E + h)^2} \neq 9.8\ \text{m/s}^2$$

This is not constant. It decreases with altitude. How is it that we can treat it as constant?

The reason can be seen by using a mathematical trick known as the binomial expansion. For any quantity ε

$$(1+\varepsilon)^2 = \left(1 + 2\varepsilon + \varepsilon^2\right)$$

However, if $\varepsilon \ll 1$, then

$$(1+\varepsilon)^2 \cong 1 + 2\varepsilon$$

In general,

$$(1+\varepsilon)^n \cong 1 + n\varepsilon$$

This means the gravitational acceleration can be approximated as

$$\begin{aligned} g' &= \frac{GM_E}{(R_E+h)^2} \\ &= \frac{GM_E}{{R_E}^2\left(1+\frac{h}{R_E}\right)^2} \\ &= \frac{GM_E}{{R_E}^2}\left(1+\frac{h}{R_E}\right)^{-2} \\ &= g\left(1+\frac{h}{R_E}\right)^{-2} \\ &\cong g\left(1-\frac{2h}{R_E}\right) \end{aligned}$$

So, for at an altitude h above the Earth's surface, the acceleration of gravity is

$$g' = g - \frac{2gh}{R_E}$$

which decreases linearly with increasing height. The radius of Earth is about 6.4×10^6 m, so this extra term will probably be pretty small, but how small?

Currently the world's tallest building is Dubai Tower. When completed in 2009 it is expected to be over 800 m tall. Just to be on the safe side, let's consider h to be 1000 m.

[8] More or less. Remember what I said about glossing over details.

From the above equation, we can see that gravity will decrease by

$$\begin{aligned}\Delta g &= \frac{2gh}{R_E} \\ &= \frac{2(9.8)(1000)}{(6.4 \times 10^6)} \\ &= 0.0030625 \text{ m/s}^2\end{aligned}$$

Very small indeed. So as long as we are within 1km of the Earth's surface, we can consider gravitational acceleration to be quite constant.

Exercises

1. A helek is a unit of time equal to 1/18 of a minute. A span is a unit of length equal to 9 inches. If a car travels with a speed of 20 m/s, what is its speed in span per helek?
   Answer: 292.

2. At what height above the Earth does the acceleration of gravity decrease by 1% of that on the Earth's surface ($\Delta g = 0.01g$)?
   Answer: About 32 km.

References

1. This lecture draws heavily from the lecture notes of Michael Richmond:
   `http://spiff.rit.edu/classes/phys311.old/phys311.html`

2. More information regarding the International System of units can be found at NIST:
   `http://physics.nist.gov/Units/current.html`