

In Honor of Arthur C. Clarke

Recently science fiction author Arthur C. Clarke passed away at his home in Sri Lanka. Clarke is perhaps best known for his work on 2001: A Space Odyssey, but his greatest work was probably his idea for a communication satellite. Originally published in the October 1945 issue of *Wireless World*, Clarke outlined how a satellite could be put in geostationary orbit. The satellite would orbit the earth about its equator once every day, and thus would always be in the same place in the sky.

To see how the idea works, consider an object moving in a circle at a constant speed, known as uniform circular motion. As the object moves around the circle with a speed v , it is constantly accelerated toward the center of the circle with an acceleration

$$a_c = \frac{v^2}{r}$$

This acceleration, known as centripetal acceleration, must be caused by a force. In the case of an object orbiting the earth, that force is gravity. For a satellite orbiting the earth in a circle of radius r , the gravitational force gives the satellite an acceleration

$$a_c = \frac{GM_e}{r^2}$$

where $G = 6.7 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$ is the universal constant of gravity and $M_e = 6 \times 10^{24} \text{ kg}$ is the mass of the earth.

This means the speed of the satellite is related to the radius of its orbit. That is,

$$\frac{v^2}{r} = \frac{GM_e}{r^2}$$

or

$$v = \sqrt{\frac{GM_e}{r}}$$

In other words, the bigger the orbital radius, the slower the satellite moves and the longer it takes to orbit the earth. This relationship can be seen more clearly by expressing the speed of the satellite to its orbital period T . Speed is distance travelled in a given time. In the time of one orbit the satellite travels one complete circle, so its speed can be written as

$$v = \frac{2\pi r}{T}$$

So this means the radius of a satellite's orbit, and the time it takes to orbit once are related by

$$\frac{2\pi r}{T} = \sqrt{G \frac{M_e}{r}}$$

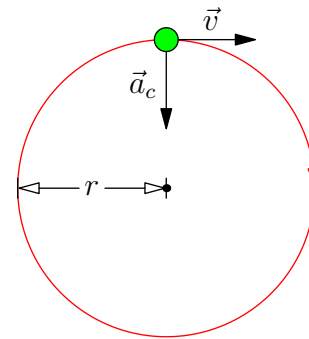


Figure 1. Uniform Circular Motion

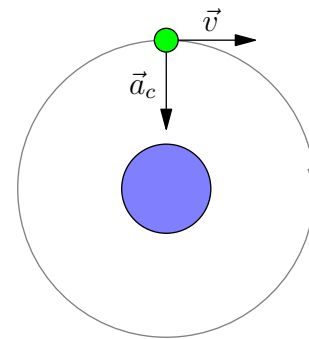


Figure 2. Circular Orbit

With a little algebra, this can be written as

$$T = \sqrt{\frac{4\pi^2 r^3}{GM_e}}$$

This means that the orbital period of a satellite depends only on its orbital radius. If we set the radius *just* right, we can ensure that the orbital period is once a day, or about 24 hours.

So what is that special radius? If we plug the numbers into our equation, we find the magic radius to be $r = 42,000 \text{ km}$, or an altitude of about 22,000 miles. Today, sixty years after Arthur C. Clarke proposed his idea, more than 300 satellites orbit the earth in geostationary orbit, also known as Clarke orbit. Which just goes to show that sometimes science fiction can become science fact.